EXAMPLE SOLUTIONS TO SAMPLE PAT PAPER

Q1: $\frac{63}{15625}$ from $(2^n - 1)/5^n$

Q2: $y = \int \left(x^2 + \frac{1}{x^3}\right) \mathrm{d}x = \frac{x^3}{3} - \frac{1}{2x^2} + D$. As $y(1) = 0$, we get that $D = 1/6$. Therefore

$$\int_1^3 y \, \mathrm{d}x = \int_1^3 \left(\frac{x^3}{3} - \frac{1}{2x^2} + \frac{1}{6}\right) \mathrm{d}x = \left[\frac{x^4}{12} + \frac{1}{2x} + \frac{x}{6}\right]_1^3 = \frac{20}{3}$$

Q3: 10

Q4: y/4

Q5: The following statements are true:

- $^{15}_{7}$N has a larger mass than $^{14}_{7}$N.
- $^{16}_{8}$O has a larger nuclear charge than $^{15}_{7}$N.
- $^{18}_{8}$O has a larger mass per unit charge than $^{12}_{6}$C.

Q6: 5 seeds

Q7: 10^4 Hz

Q8: $-2f(x - 1)$

Q9: $16 + 12x + 3x^2 + \frac{x^3}{4}$

Q10: 1.2 m s^{-1}

Q11: a, c. It can be found that b, d is also correct but this is not an option.

Q12: $2F$. Since $\frac{1}{2}mv^2 = Fd$ to stop it, and we need $F^{\text{circle}} = \frac{mv^2}{d}$, we find that $F' = 2F$.

Q13: $P = I_D V_D$ and $NI_D = 20NI_S$. Putting numbers in we get $I_S = 2.5A$.

Q14: Reflection in $y = 0$. Note that answer C produces a triangle in the desired position but has points B and C swapped and is thus incorrect.

Q15: The answer is D because it is placed in the branch with the lowest total resistance of $4/3R$.

Q16: $x = \frac{1}{2}$. Whilst $x = -2$ is a solution to the quadratic form that arises after exponentiating the equation $(2x^2 + 3x - 2 = 0)$ it is not a valid argument for the $\log_2$ function.

Q17: $R/\sqrt{5}$. The field strength is proportional to $1/r^2$ so that $g_E = k/R_E^2$ and $g_R = k/R^2$ with the same value of k.

Q18: There are 3 such maxima. $\sin(\alpha)$ has maxima for $\alpha \in (90, 450, 810, 1170, ...)$. The first three are accessible for $x > 0.1$ so the answer is **C**

Q19: Ordered in increasing wavelength:
(1) radioactive source (gamma rays)
(2) electric torch (white light)
(3) hot cooking stove (infrared)
(4) microwave oven (microwaves)
(5) short-wave radio transmitter (radio)

Q20: The different possible decay final states of two X particles which are compatible with the observation of at least one pair of Y particles are: (YY, YY), (YY, ZZ), (ZZ, YY) each of which has equal probability of 1/3. In two of these outcomes a pair of Z particles is present so the answer is $P = 2/3$.

Q21: We need a total of $4*3+4*2+2*1 = 22$ practicals which can be arranged in 4 sessions as follows:
Session 1: 4 students needing three practical + 2 students needing 2 practical
Session 2: 4 students needing three practical + 2 students needing 2 practical
Session 3: 4 students needing three practical + 2 students needing 1 practical
Session 4: 4 students needing 2 practical

Q22: The sequence is one of prime numbers and the next prime is 61

Q23: Atoms have typical dimensions $R_{\text{atom}} \approx 10^{-10}\,\text{m}$ and hence volume $V_{\text{atom}} = 10^{-30}\,\text{m}^3$. The dimension of our stone is $L_0 = 10^{-1}\,\text{m}$ and hence $V_{\text{stone}} = 10^{-3}\,\text{m}^3$. After the n$^{\text{th}}$ hit the dimension of the stone is $V_n = \frac{10^{-3}}{3^n}$ so we demand $\frac{10^{-3}}{3^n} \leq 10^{-30}$ which leads to $n \leq \log_3(10^{30-3}) = \frac{\ln(10^{27})}{\ln(3)} \approx 56.6$.

Q24: Answer is E. The negative sign in front of f mirrors the function along the x-axis. Inverting just the sign of x in the argument shifts the peak to be at $-b$. Adding a to the argument shifts the graph to the right by less than b so it now peaks at $(a-b)$.

Q25: The next number is 4. Each number is equal to its precursor divided by -3.

Q26: After exponentiating $xy^2 = 32$ and $\frac{x}{y} = \frac{1}{2}$. Insert $x = y/2$ from the second equation into the first gives $y^3 = 64$ which yields $y = 4$. Inserting this back into the first equation gives x=2.

Q27: Gravitational potential energy is proportional to r^{-1} and hence $n = -1$ The total Energy is the sum of potential and kinetic energy

$$\langle E_{\text{tot}} \rangle = \langle V_{\text{tot}} \rangle + \langle T_{\text{tot}} \rangle = \langle V_{\text{tot}} \rangle - \frac{1}{2}\langle V_{\text{tot}} \rangle = \frac{1}{2}\langle V_{\text{tot}} \rangle$$

Q28: $\frac{g_{\text{planet}}}{g_{\text{Earth}}} = 2$.

Q29: $\alpha \leq -2$ or $\alpha \geq 2$. Using trig relations the equation can be rephrased as $\tan^2\theta + \alpha\tan\theta + 1 = 0$. This is a quadratic in $\tan\theta$ with the solution: $\tan\theta = \frac{-\alpha \pm \sqrt{\alpha^2-4}}{2}$ which requires $\alpha^2 \geq 4$.

Q30: $\left(\frac{2br}{(b+r)(b+r-1)}\right)$ — the sum of the probabilities for the two possible positive outcomes which are br and rb.

Q31: $N_{\text{max}} = 2^{10} = 1024$ since we can use our fingers to indicate the digits of a 10 digit binary number.

Q32: $I_2 = -I$. The B field changes its direction when the current changes its direction. Both wires will contribute the same field at the middle line and hence $I_2 = -I$.

Q33: New moon.

Q34: (1) only B closed (2) only A closed (3) both closed.

Q35: $L = \frac{v}{4f_{\text{min}}}$. The standing wave in the pipe has to have a node at the closed end and a maximum at the open end with no other nodes between. This means that the length of the pipe $L = \lambda_{\text{max}}/4$ and $L = v/(4f_{\text{min}})$.

Q36: $\mathrm{C\,m\,s^{-1}}$. In SI base units (m, s, mol, A, K, cd, Kg) all other combinations give $\mathrm{Kg\,m\,s^{-2}} = \mathrm{N}$

Q37: Twice right and once left. $P = 12/27 = 4/9$.

Q38: This is the sum of a geometric progression to infinity with an unknown ratio α. We want the sum $S = \frac{1}{1-\alpha} \leq 3$ therefore $\alpha \leq \frac{2}{3}$

Q39: The area of the roof is $A = 100 \cdot 50\,\mathrm{m}^2 = 5000\,\mathrm{m}^2$. The total mass is $m = 5000 \times 100\,\mathrm{kg}$ so the weight is $F = mg = 9.81 \cdot 5 \cdot 10^5$ N$= 4.9 \cdot 10^6$ N.

Q40: $y = -\frac{1}{2}x + \frac{1}{2}$. The line needs to have a gradient of $\frac{-1}{a}$ where a is the gradient of the line it intersects perpendicularly. It also needs to go through the point $(x = 1, y = 0)$